Turbulence modeling of wind fields

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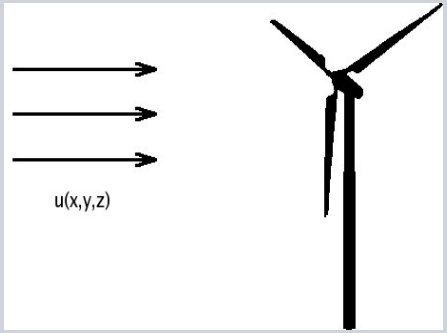
Corporate Research
& Technology
+ U Giessen
+ U Århus

External wind conditions affect structural loading, durability and operation of wind turbines.

(1) ambient turbulence,
(2) wake turbulence.



Three types of wind traces
with <u>=10m/sec and 15% turbulence intensity

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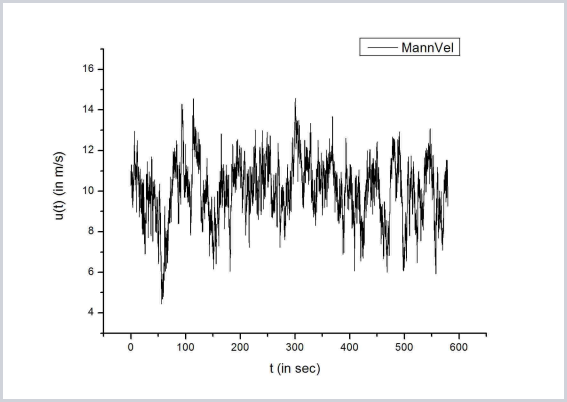
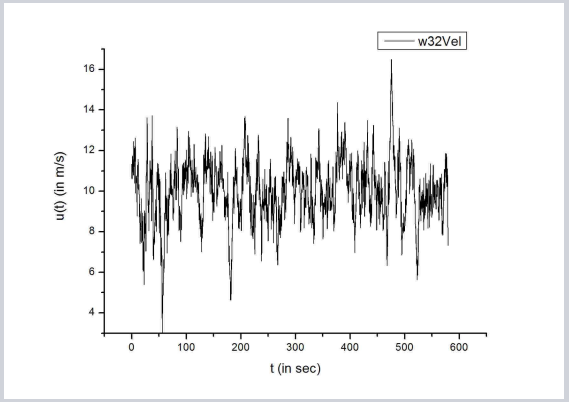
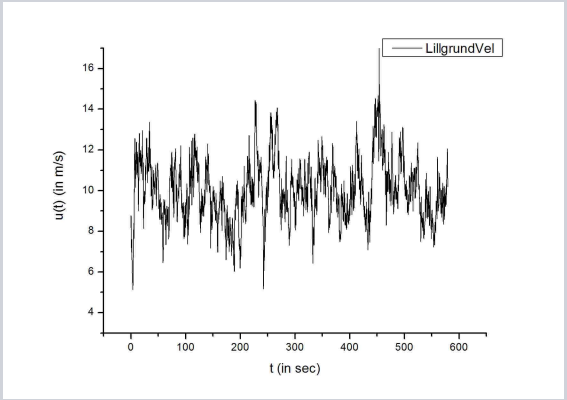


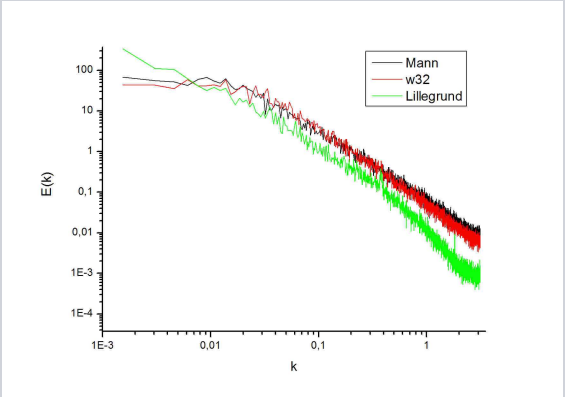
Mann model (L=64m)

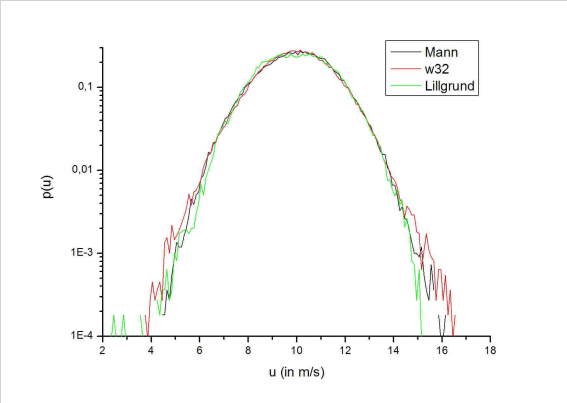


Multifractal model (L=64m)



Lillegrund data (L=250m)





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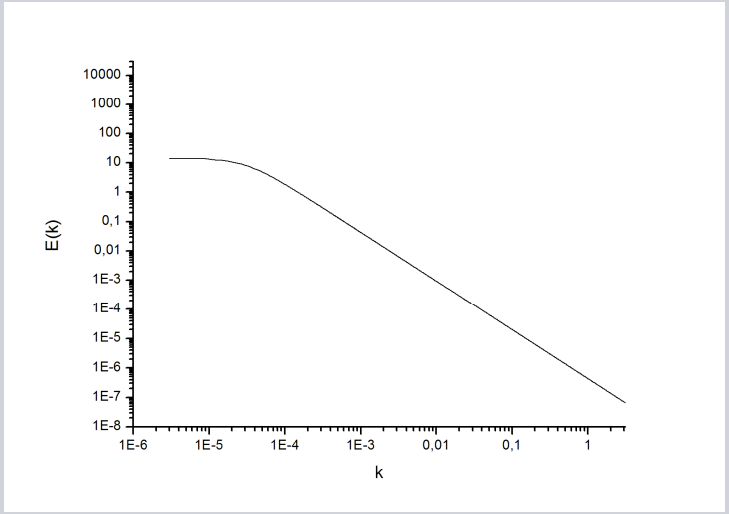
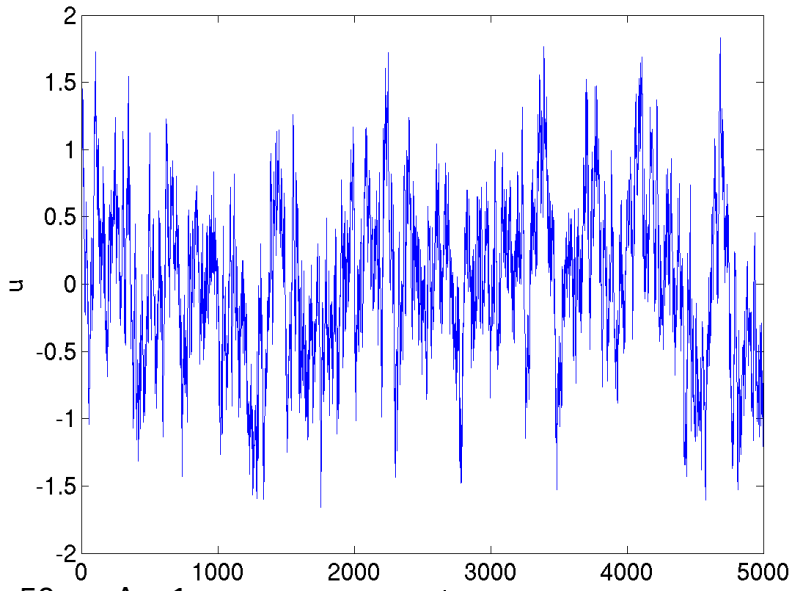
Standard (IEC 61400) ambient turbulence modeling I

$$E(k) = C \Big/ \left(1 + L^2 k^2\right)^{5/6}, \qquad u(x \text{ or } t) = \sum_k \sqrt{E(k)} n_k e^{ikx}$$



von Karman spectrum: 5/3 law

time trace

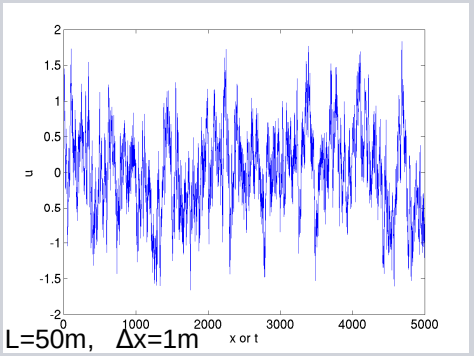


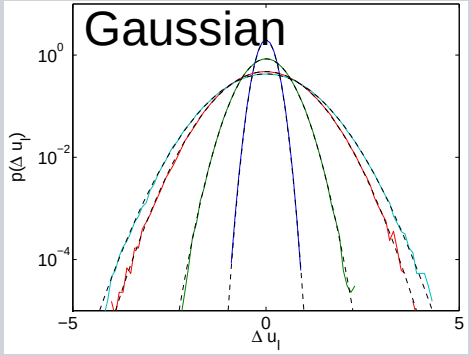
L=50m, Δx=1m

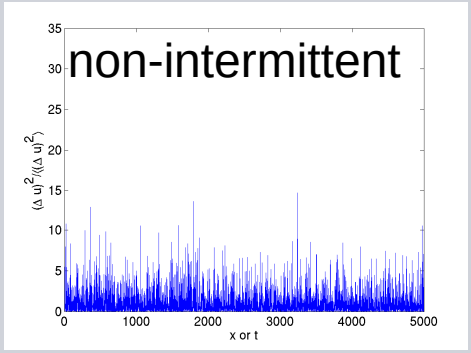
Standard ambient turbulence modeling II

$$E(k) = C\Big/\left(1 + L^2k^2\right)^{5/6}, \qquad u(x \text{ or } t) = \sum_k \sqrt{E(k)n_k}\, e^{ikx}$$

standard normal turbulence model





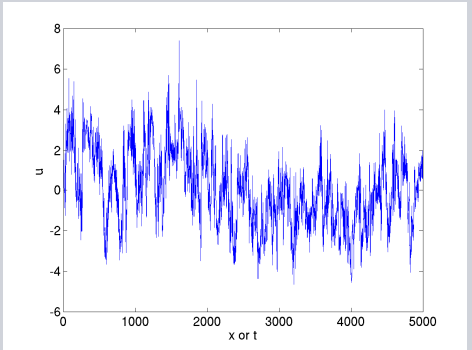


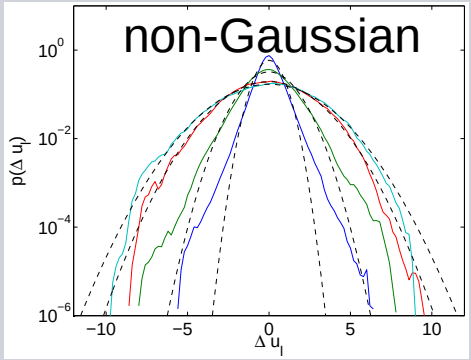
$$\Delta u_l(x) = u(x+l) - u(x) \qquad \Delta u(x)^2 = \left[u(x+\Delta x) - u(x)\right]^2$$

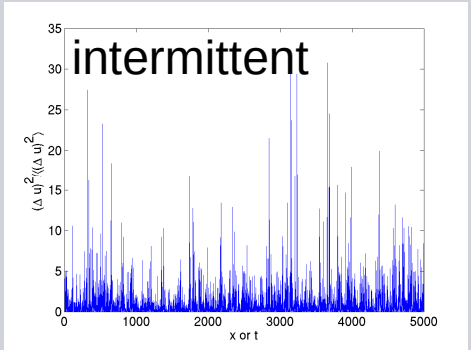
l = 1024m, 32m, 8m, 1m

$$\Delta x = 1m$$

observed atmospheric boundary layer







Multifractal extension of standard ambient turbulence modeling I

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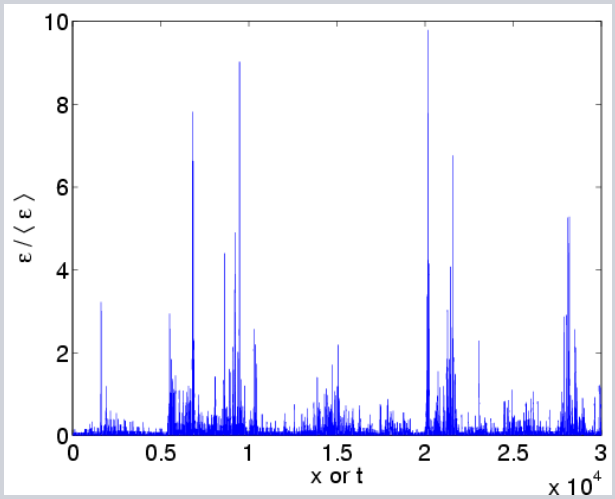
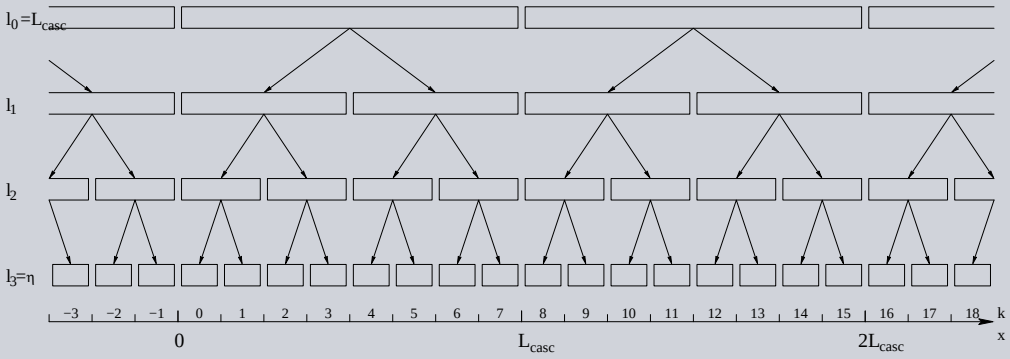
$$E(k) = C \Big/ \left(1 + L^2 k^2\right)^{5/6}$$

$$u(x) = \sum_k \sqrt{E(k)}\, n_k e^{ikx}$$

$$\Delta u(x) = u(x + \Delta x) - u(x)$$



$$\Delta u(x) \Rightarrow \left(\text{multifractal}(x)\right)^{1/3} \Delta u(x)$$



multifractal energy cascade

$$\pi(l) = \Delta u(l)^3 / l$$

Novikov'71
Benzi+Paladin+
Parisi+Vulpiani'84

hierarchy of scales

$$L \to L/\lambda \to \cdots \to L/\lambda^j \to \cdots \to L/\lambda^J = \eta$$

random multiplication

$$m = q_1 \cdot q_2 \cdots q_j \cdots q_J$$

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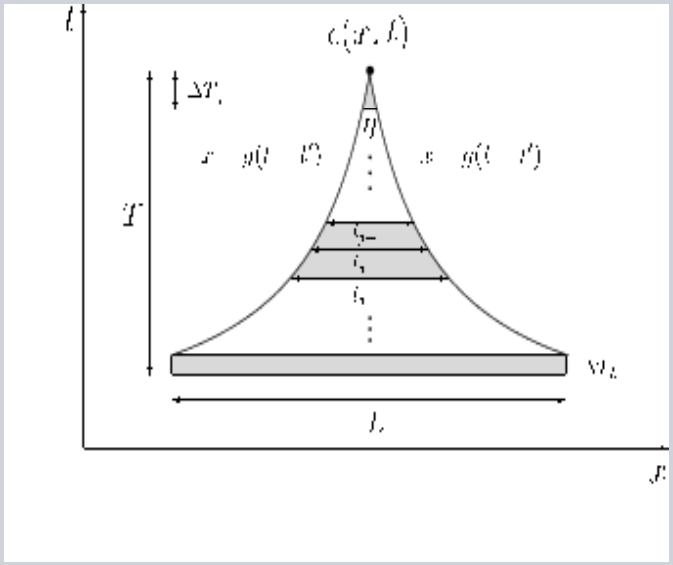
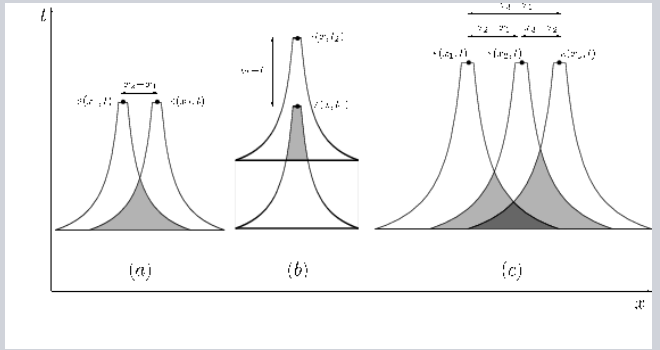
more multifractal modeling

$$m(x,t) = \exp\left(\int_{t-T}^{t} dt' \int_{x-L/2}^{x+L/2} dx' \, f(x-x', t-t') \, \gamma(x', t') \right)$$



PLA 320 (2004) 247
EPJB 63 (2008) 109



Multifractal extension of standard ambient turbulence modeling II

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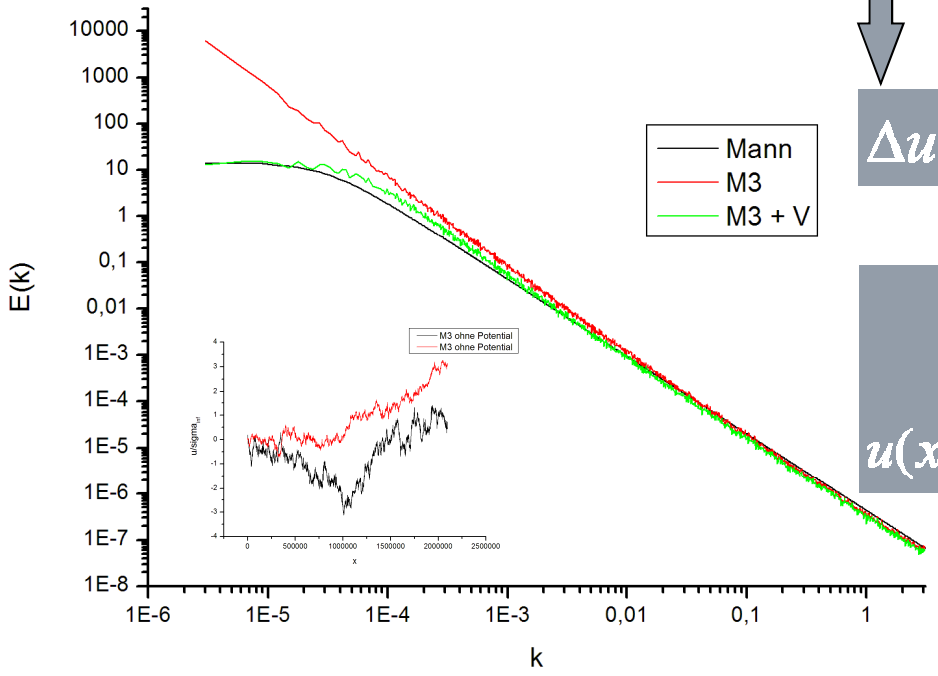

$$E(k) = C \Big/ \left(1 + L^2 k^2\right)^{5/6}$$

$$u(x) = \sum_k \sqrt{E(k)}\, n_k\, e^{ikx}$$

$$\Delta u(x) = u(x + \Delta x) - u(x)$$

$$\Delta u(x) \Rightarrow \left(\text{multifractal}(x)\right)^{1/3} \Delta u(x)$$

$$\Delta u(x) \Rightarrow -\frac{\partial V(u)}{\partial u} + \sqrt[3]{\text{m}(x)}\, \Delta u(x)$$

$$u(x + \Delta x) = u(x) + \Delta u(x)$$

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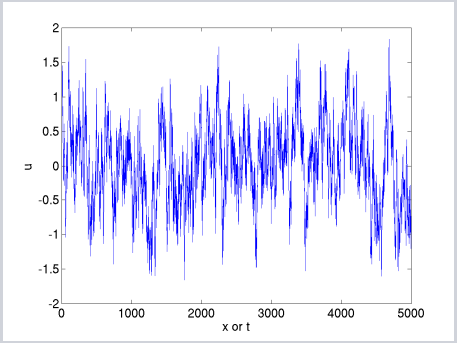
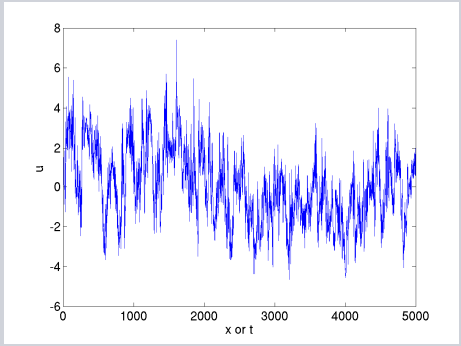
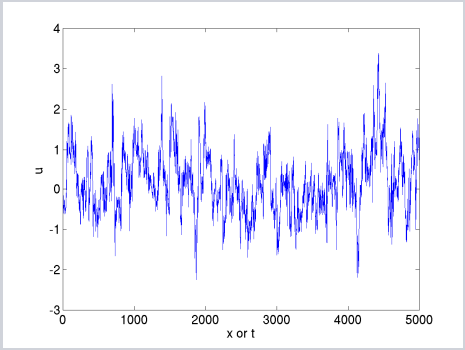
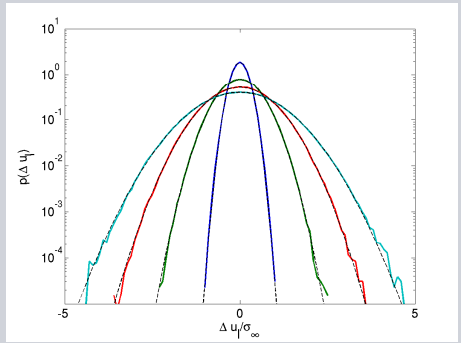
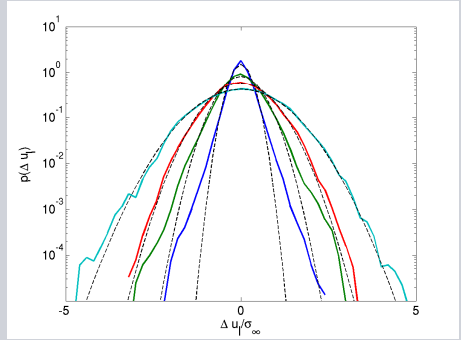
Wind traces: standard / real / multifractal

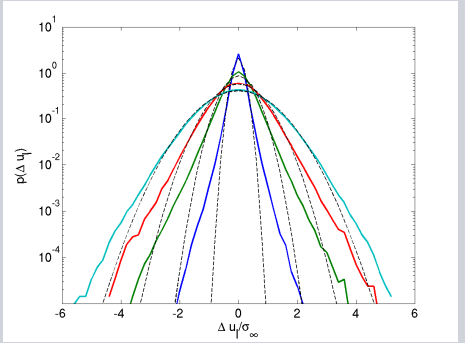


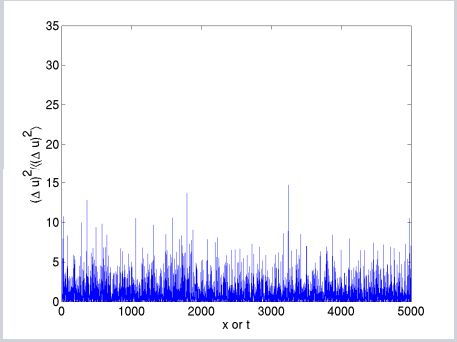


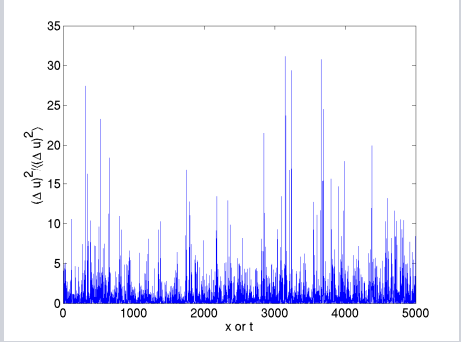


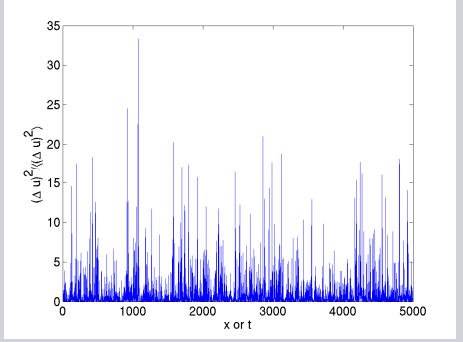






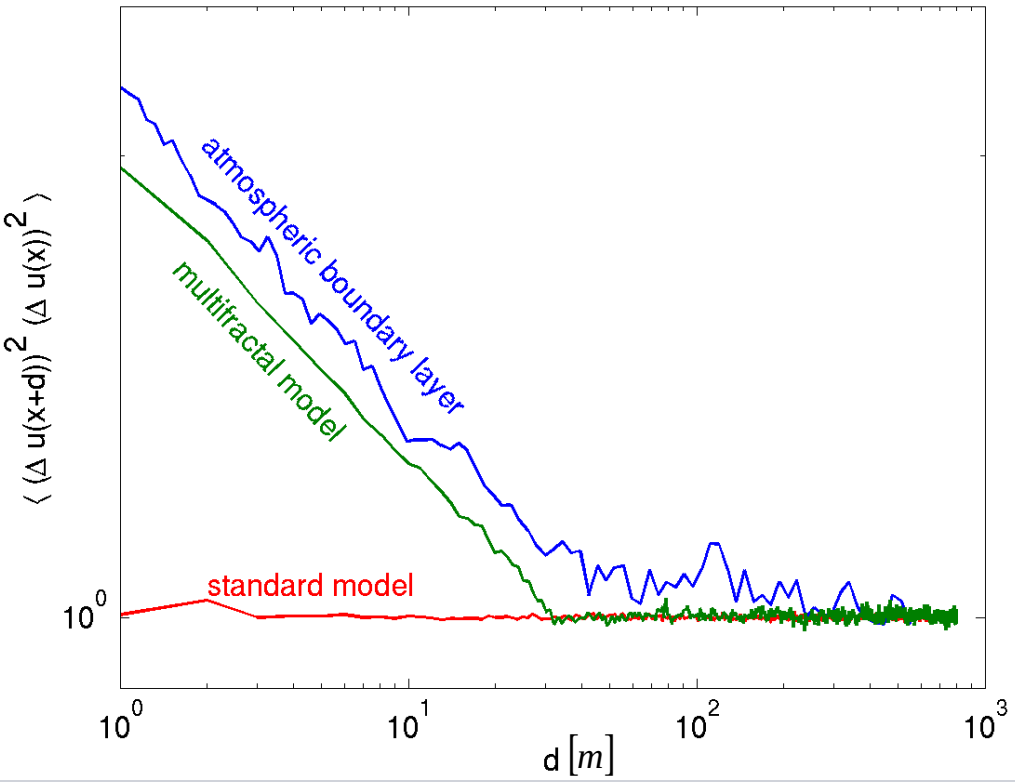






Wind gust correlations (onshore)

$$\left\langle \Delta u(x+d)^2 \, \Delta u(x)^2 \right\rangle \quad \propto \quad d^{-\mu}$$

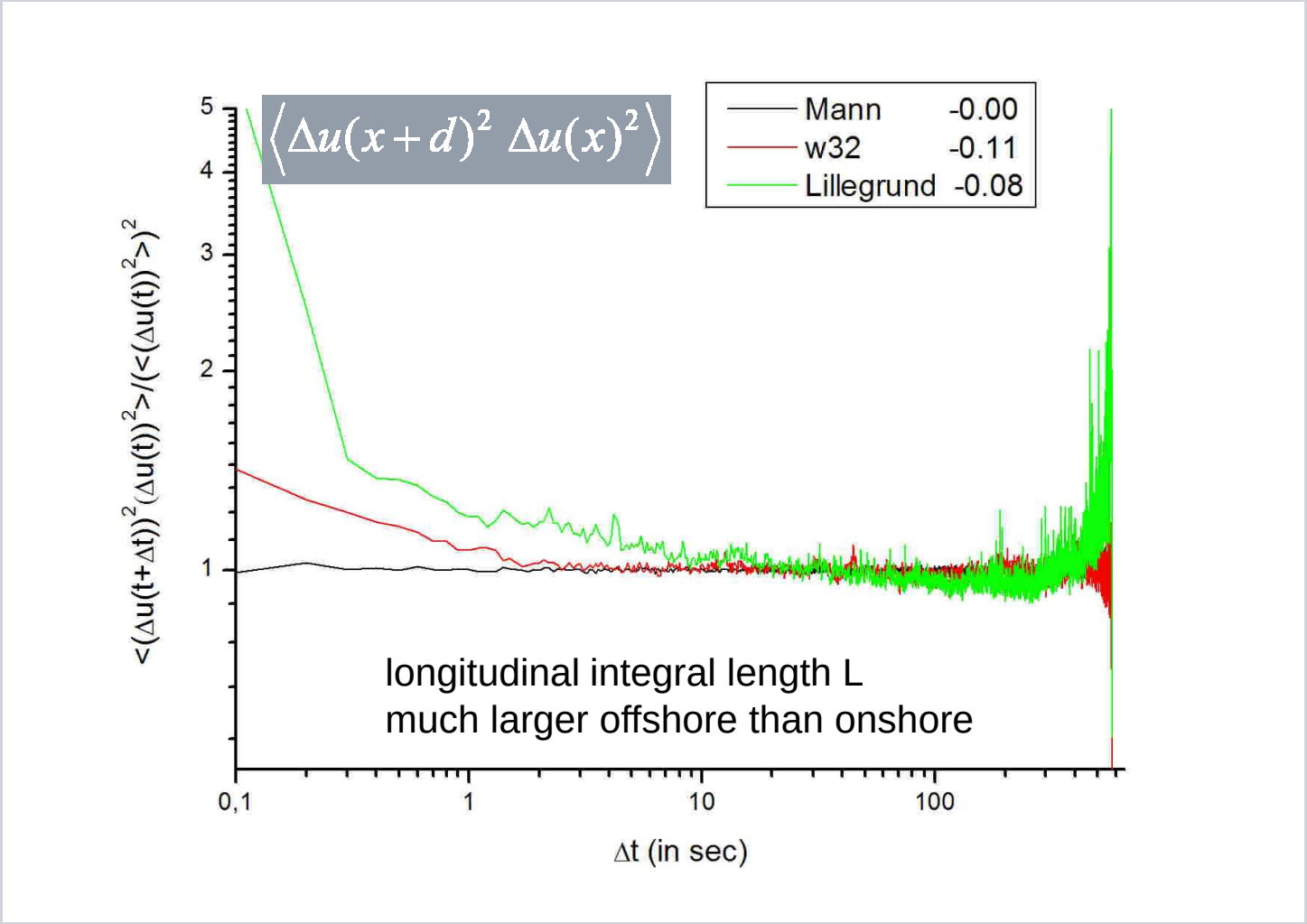


multifractal fingerprint μ

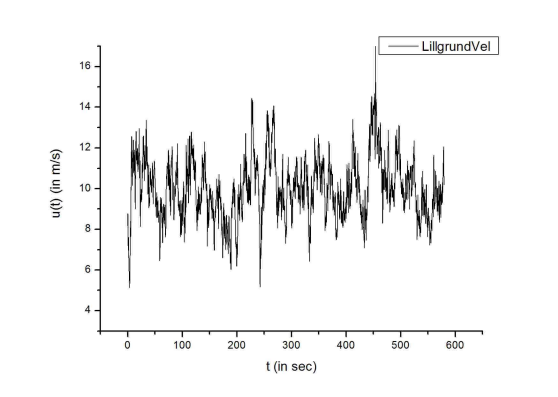
correlation length ≈ 30-50m,
correlation time ≈ 5-10sec

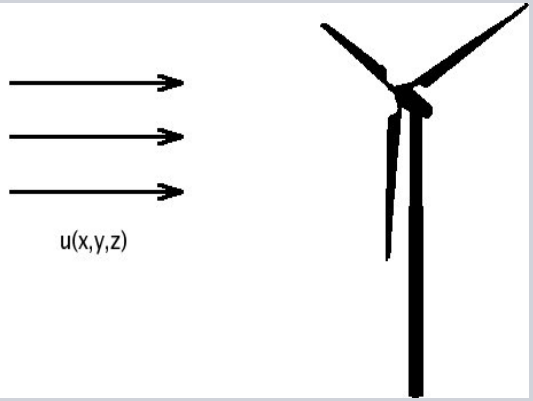
length of rotor blade ≈ 40-50m

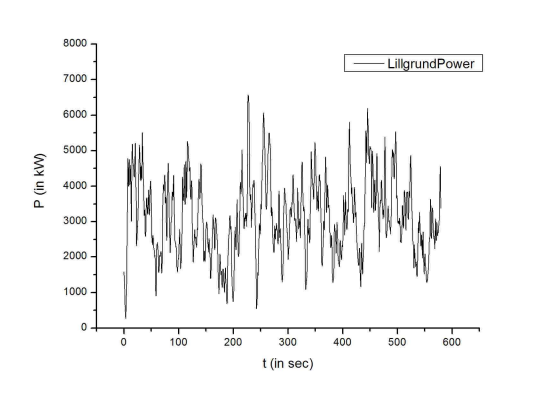
Wind gust correlations (offshore)



Wind-turbine interaction I







Wind-turbine interaction II:
power autocorrelation ➔ integral length L

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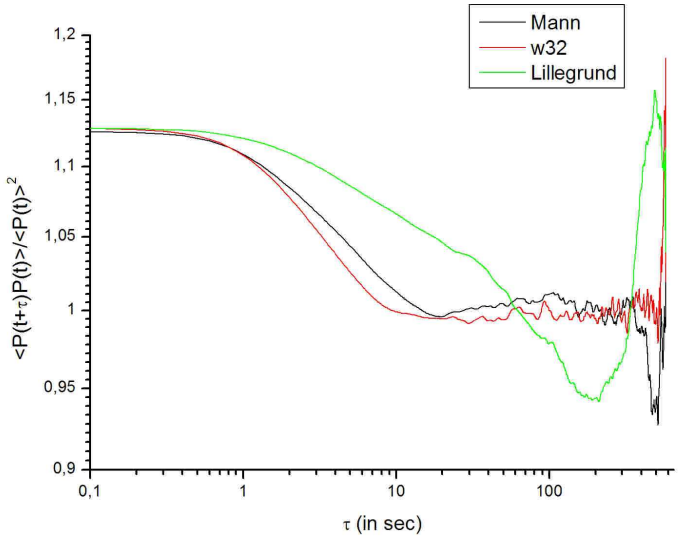

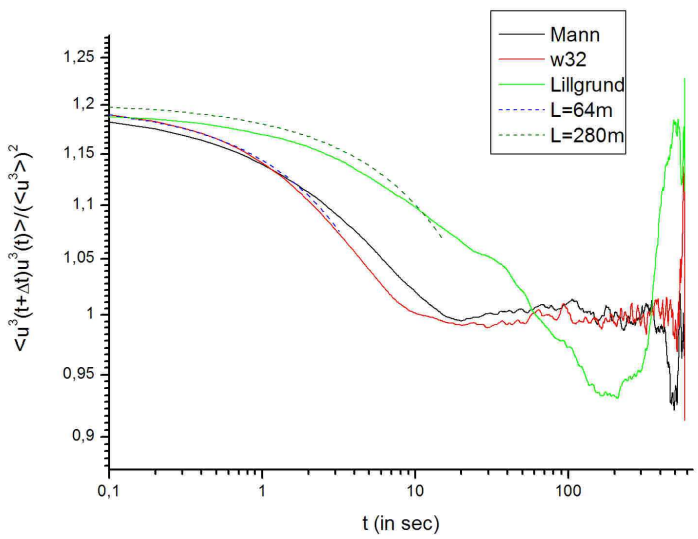

$$\frac{\langle P(t+\Delta t)P(t)\rangle}{\langle P\rangle^2}$$

$$\frac{\langle u^3(x+l)u^3(x)\rangle}{\langle u^3\rangle^2} \approx 1+9\frac{\sigma^2}{u^2}\left(1-\left(\frac{l}{L}\right)^{2/3}\right)$$

longitudinal integral length L
much larger offshore than onshore

Wind-turbine interaction III:
power-gradient autocorrelation ➔ multifractality

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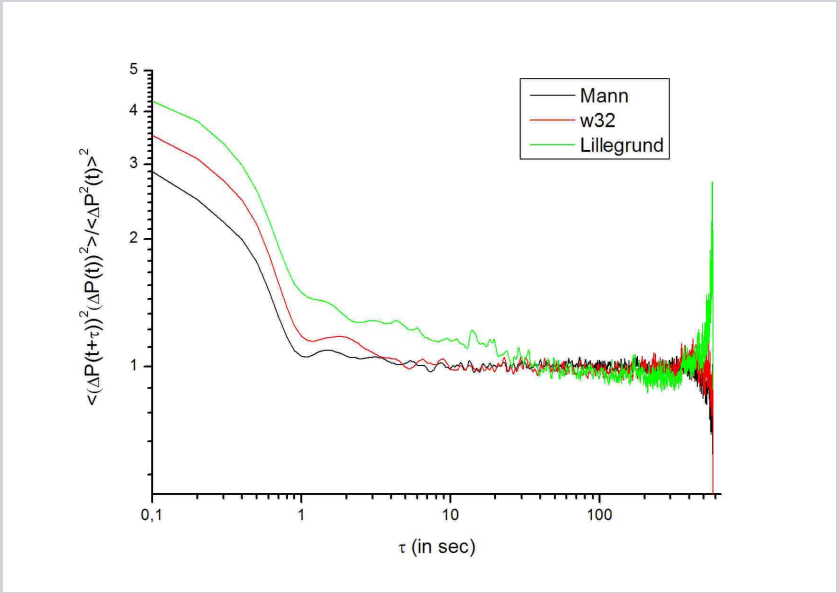

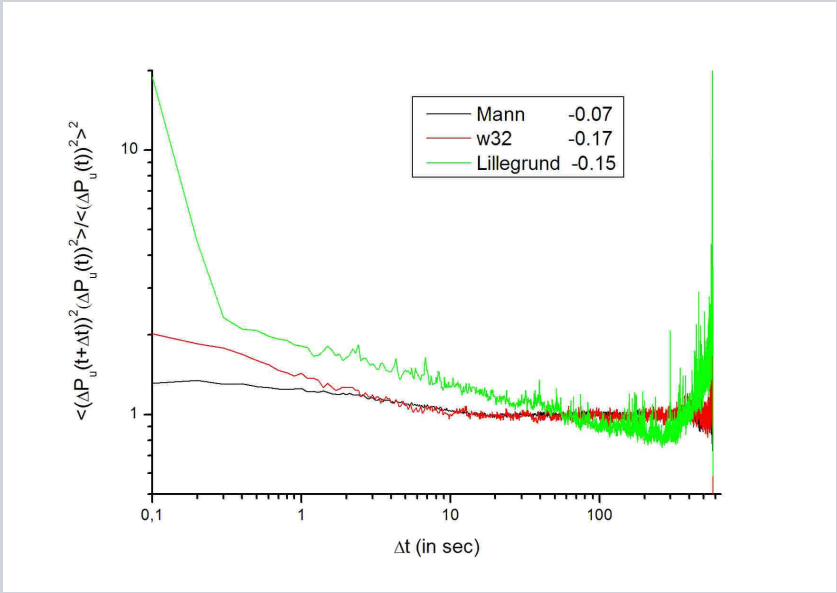

$$\frac{\left\langle \Delta P^2(t+\tau)\Delta P^2(t)\right\rangle}{\left\langle \Delta P^2\right\rangle^2}$$

$$\Delta P(t) = P(t+\tau) - P(t)$$

$$\frac{\left\langle \Delta P_u^2(t+\Delta t)\Delta P_u^2(t)\right\rangle}{\left\langle \Delta P_u^2\right\rangle^2}$$

$$\Delta P_u(t) = u^3(t+\tau) - u^3(t)$$

Wind-turbine interaction IV: multifractal fingerprint (μ)
wind-gust correlation ➔ power-gradient correlation

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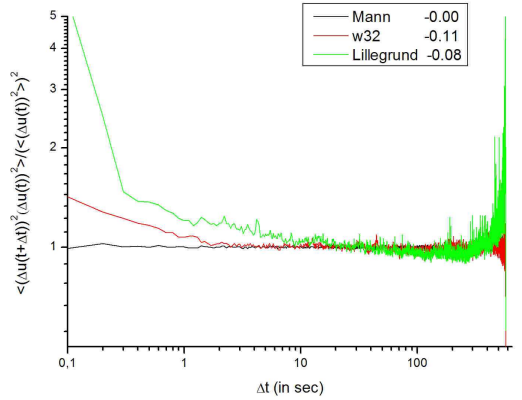
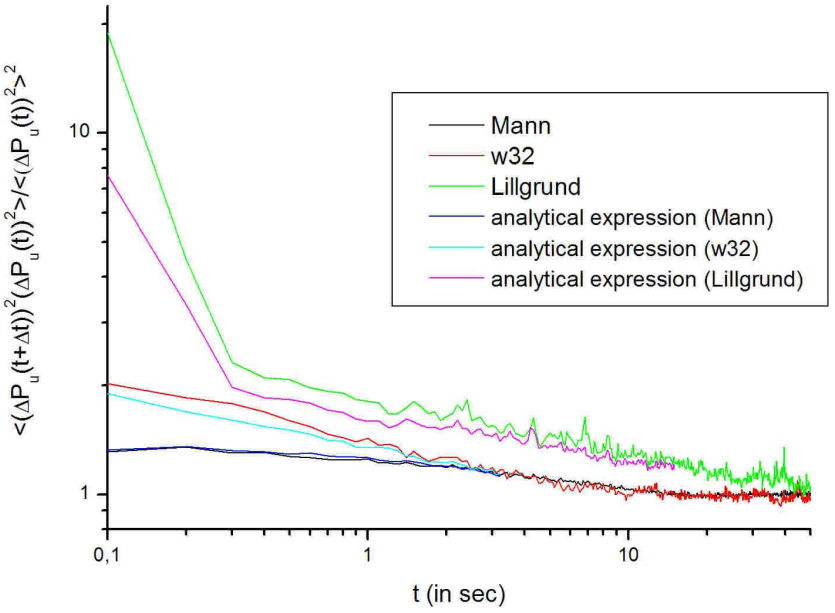
$$\left\langle \Delta P_u^{\,2}(x+l)\Delta P_u^{\,2}(x)\right\rangle / \left\langle \Delta P_u^{2}\right\rangle^2$$
$$\approx 1+16\frac{\sigma^2}{u^2}\left(1-\left(\frac{l}{L}\right)^{2/3}\right)\frac{\left\langle \Delta u^2(x+l)\Delta u^2(x)\right\rangle}{\left\langle \Delta u^2\right\rangle^2}$$





More multifractal modeling: from wind traces to wind fields

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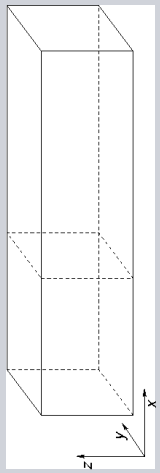
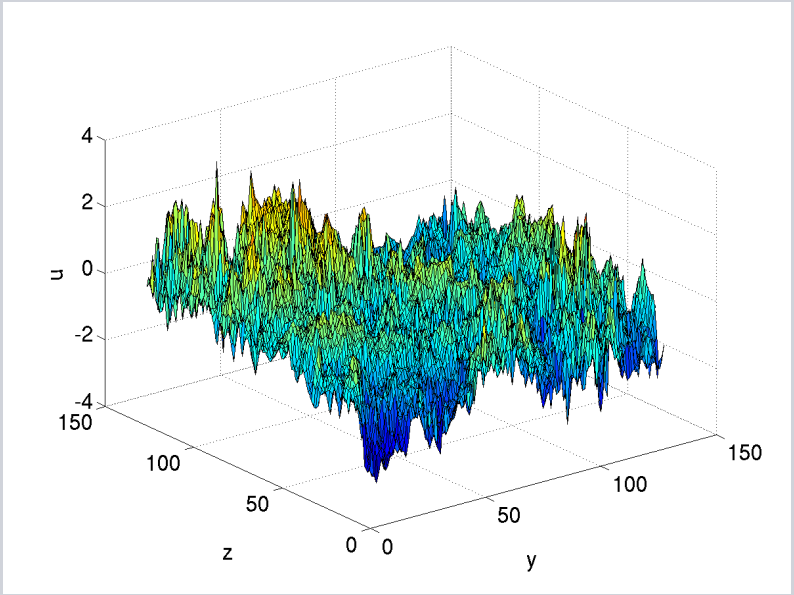
$u(x, y, z)$





ambient turbulence
= multifractal turbulence

→ turbine loads
→ blade design
→ new standard

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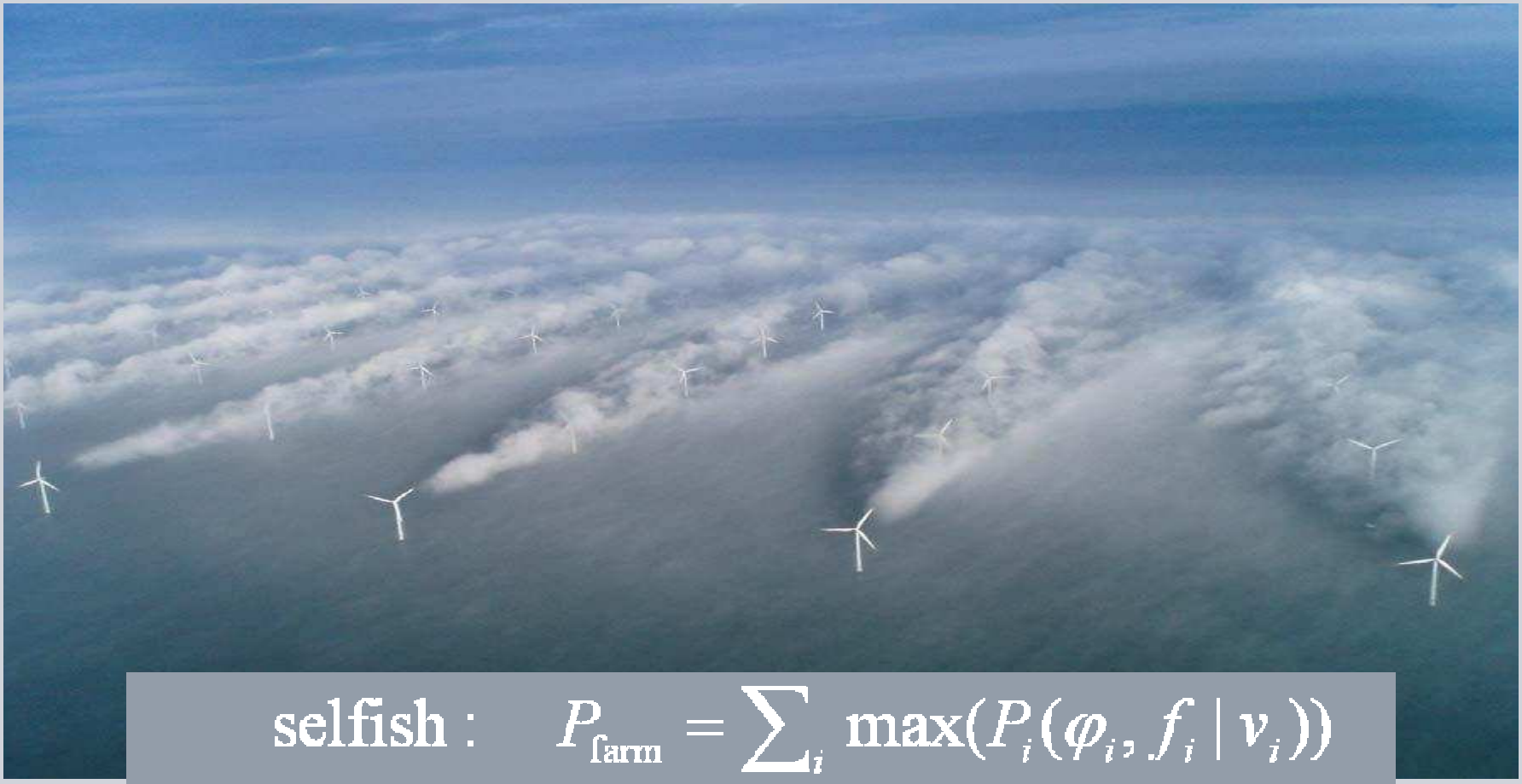
Wind farm optimization + control



$$\text{selfish}: \quad P_{\text{farm}} = \sum_i \max(P_i(\varphi_i, f_i \mid v_i))$$

$$\text{cooperative}: \quad P_{\text{farm}} = \max\left(\sum_i P_i(\varphi_i, f_i \mid v_i, \theta)\right)$$