Stochastic modeling of electricity prices
– a survey –

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Ambit processes, non-semimartingales and applications,
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Overview

1. Goal: Motivate the use of ambit processes
2. Introduction to electricity markets
3. Stylized facts of electricity prices
4. Stochastic modelling of electricity prices

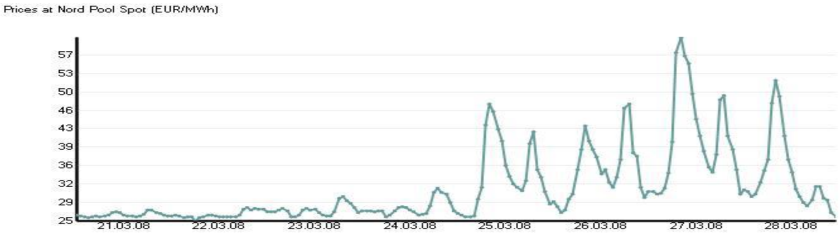
The NordPool Market

- The NordPool market organizes trade in
  - Hourly spot electricity, next-day delivery
  - Forward and futures contracts on the spot
  - European options on forwards
- Covers the Nordic region
  - Norway, Sweden, Denmark and Finland
  - Northern Germany
- Power production
  - Hydro, nuclear, coal, wind

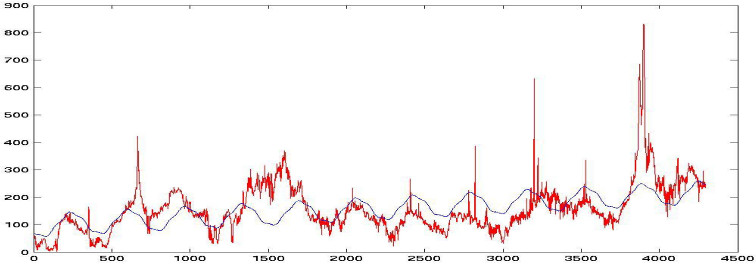
Elspot: the spot market

- A (non-mandatory) hourly market with physical delivery of electricity
- Participants hand in bids before noon *the day ahead*
    - Volume and price for each of the 24 hours next day
    - Maximum of 64 bids within technical volume and price limits
- NordPool creates demand and production curves for each hour of the next day

- The system price is the equilibrium
  - Price for delivery of electricity at a specific hour next day
  - The *daily* system price is the average of the 24 hourly
- Reference price for the forward market
- A series of hourly prices from Friday 21–Friday 28 March, 2008

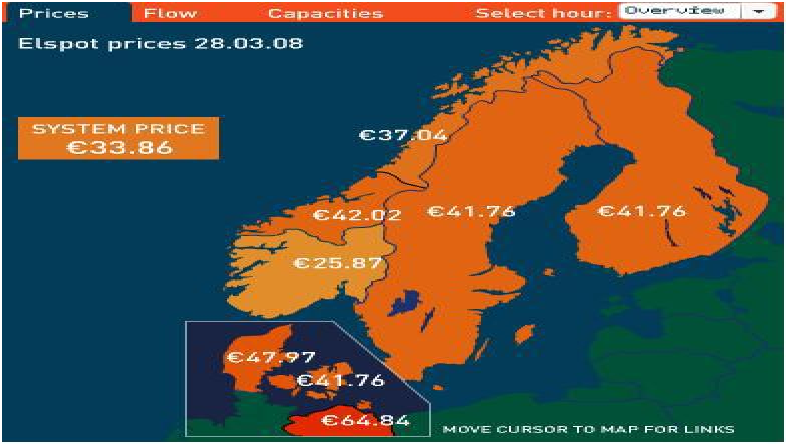


- Historical system price from the beginning in 1992 (NOK/MWh)



- Due to congestion (non-perfect transmission lines), *area prices* are derived
  - Sweden, Finland and Northern Germany separate areas
  - Denmark split into two
  - Norway may be split into several areas
- The area prices are the actual prices for the consumers/producers in the area in question

- Areas and area prices on March 28, 2008

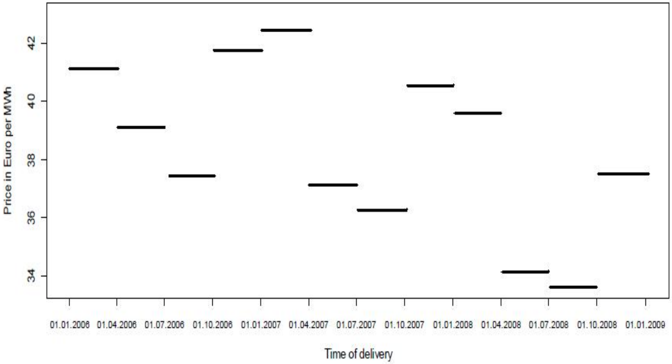


The forward and futures market

- Contracts with “delivery” of electricity over a period
  - Financially settled: The money-equivalent of receiving electricity is paid to the buyer
  - The reference is the hourly system price in the delivery period
  - Note: many markets have physical delivery of electricity
- Delivery periods
  - Next day or week (futures-style)
  - Monthly
  - Quarterly (earlier seasons)
  - Yearly
- Overlapping delivery periods (!)

- Base load contracts
  - Delivery over the whole period
- Peak load contracts
  - Peak hours are from 8 to 20 every day
  - Weekends excluded
- Also here the futures-style contracts have short delivery period
- Contracts frequently called *swaps*
  - Fixed for floating spot price

- The forward curve 1 January, 2006 (base load quarterly contracts)
- Constructed from observed prices og various delivery length



The option market

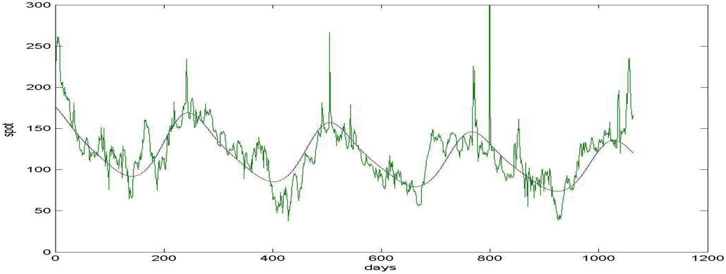
- European call and put options on electricity forwards
  - Quarterly and yearly delivery periods
- Low activity on the exchange (Option prices May 4, 2006)

Exchange quotation and trading of Power Options
Prices in EUR
Trading day: 04.05.06
Updated at hour: 15:46:06

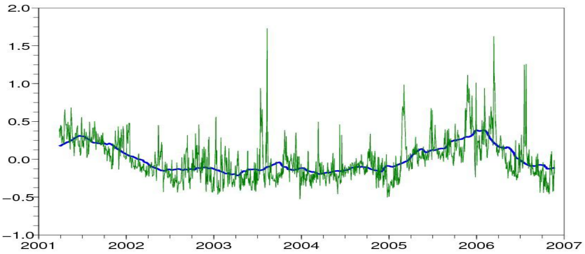
Product	Curr.	Hours	Last trading day	Best buyer	Best seller	Last traded	Change close	MW traded	Highest traded	Lowest traded	Closing price	Open interest 03.05
European												
ENOC31Q3-06	EUR	2208	14.06.06	-	-	-	-0.44	-	-	-	11.04	0
ENOC32Q3-06	EUR	2208	14.06.06	-	-	-	-0.44	-	-	-	10.07	0
ENOC33Q3-06	EUR	2208	14.06.06	-	-	-	-0.43	-	-	-	9.13	0
ENOC34Q3-06	EUR	2208	14.06.06	-	-	-	-0.43	-	-	-	8.20	0
ENOC35Q3-06	EUR	2208	14.06.06	-	-	-	-0.42	-	-	-	7.31	0
ENOC36Q3-06	EUR	2208	14.06.06	-	-	-	-0.40	-	-	-	6.46	0
ENOC37Q3-06	EUR	2208	14.06.06	-	-	-	-0.39	-	-	-	5.65	0
ENOC38Q3-06	EUR	2208	14.06.06	-	-	-	-0.37	-	-	-	4.90	0
ENOC39Q3-06	EUR	2208	14.06.06	-	-	-	-0.35	-	-	-	4.21	0
ENOC40Q3-06	EUR	2208	14.06.06	-	-	-	-0.32	-	-	-	3.60	0
ENOC41Q3-06	EUR	2208	14.06.06	-	-	-	-0.30	-	-	-	3.05	0
ENOC42Q3-06	EUR	2208	14.06.06	-	-	-	-0.27	-	-	-	2.56	0
ENOC43Q3-06	EUR	2208	14.06.06	-	-	-	-0.25	-	-	-	2.13	75
ENOC44Q3-06	EUR	2208	14.06.06	-	-	-	-0.22	-	-	-	1.76	55
ENOC45Q3-06	EUR	2208	14.06.06	-	-	-	-0.19	-	-	-	1.44	0
ENOC46Q3-06	EUR	2208	14.06.06	-	-	-	-0.17	-	-	-	1.17	100
ENOC47Q3-06	EUR	2208	14.06.06	-	-	-	-0.15	-	-	-	0.95	185
ENOC48Q3-06	EUR	2208	14.06.06	-	-	-	-0.13	-	-	-	0.77	75
ENOC49Q3-06	EUR	2208	14.06.06	-	-	-	-0.11	-	-	-	0.62	0
ENOC50Q3-06	EUR	2208	14.06.06	-	-	-	-0.09	-	-	-	0.50	50
ENOC51Q3-06	EUR	2208	14.06.06	-	-	-	-0.07	-	-	-	0.40	0
ENOC52Q3-06	EUR	2208	14.06.06	-	-	-	-0.07	-	-	-	0.31	0
ENOC53Q3-06	EUR	2208	14.06.06	-	-	-	-0.05	-	-	-	0.25	50
ENOC54Q3-06	EUR	2208	14.06.06	-	-	-	-0.05	-	-	-	0.19	25
ENOC55Q3-06	EUR	2208	14.06.06	-	-	-	-0.04	-	-	-	0.15	75
ENOC56Q3-06	EUR	2208	14.06.06	-	-	-	-0.03	-	-	-	0.12	150
ENOC57Q3-06	EUR	2208	14.06.06	-	-	-	-0.03	-	-	-	0.09	0
ENOC60Q3-06	EUR	2208	14.06.06	-	-	-	-0.01	-	-	-	0.05	75
ENOP31Q3-06	EUR	2208	14.06.06	-	-	-	0.00	-	-	-	0.03	0
ENOP32Q3-06	EUR	2208	14.06.06	-	-	-	0.01	-	-	-	0.06	0

Stylized facts of electricity prices

- Seasonality on different time scales
  - Yearly
  - Weekly
  - Intra-daily
- Plot of NordPool system (spot) price



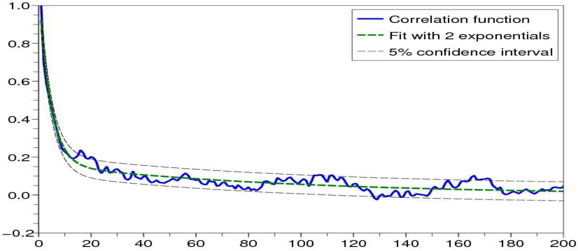
- Mean-reversion of spot prices
    - Energy prices driven by supply and demand
    - Prices will revert towards an equilibrium level
- However, to what level?
    - A fixed long-term level?
    - A stochastic level?
- Plot of UK PX log-spot prices with running mean



- Mean reversion shows up in the autocorrelation function (ACF)
  - Assuming stationarity in prices

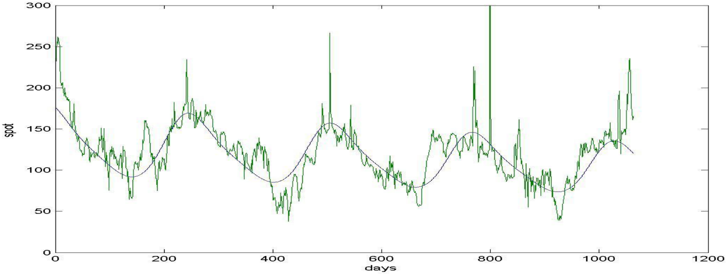
$$\rho(\tau) = \mathrm{corr}\left(S(t+\tau), S(t)\right)$$

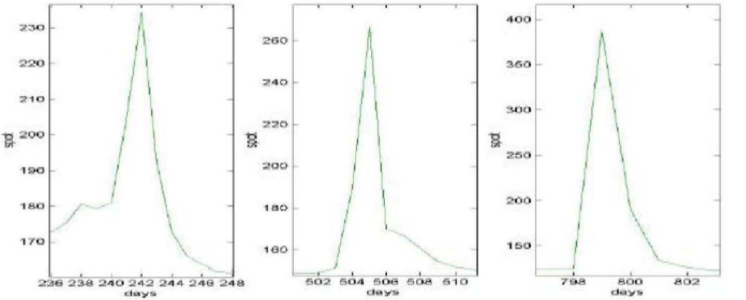
- Empirically, ACF's are often representable as sums of exponentials,
- This means that we have several scales of mean-reversion
  - Fast due to spikes
  - Medium and slow due to "normal" price variations
- Points towards several mean-reversion factors in dynamics



- Empirical ACF of EEX spot prices
  - Fitted with a sum of two exponentials
  - Multi-scale mean-reversion

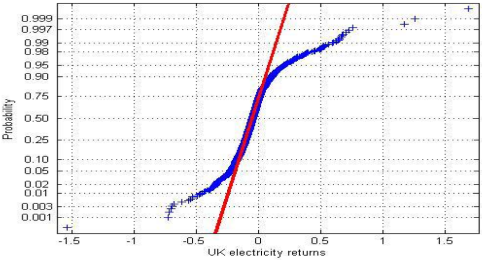
- Spikes in spot electricity
- Spike: A large price increase followed by a rapid reversion back to normal levels
    - Happens within 2-3 days
    - May be of several magnitudes
- Nord Pool price series





- Zoom-in of the three biggest spikes in NordPool series
- Note the rapid reversion, and magnitude of the increase
- Spikes occur in winter at Nord Pool
    - Other markets may not have seasonality in spike occurence (e.g. EEX)

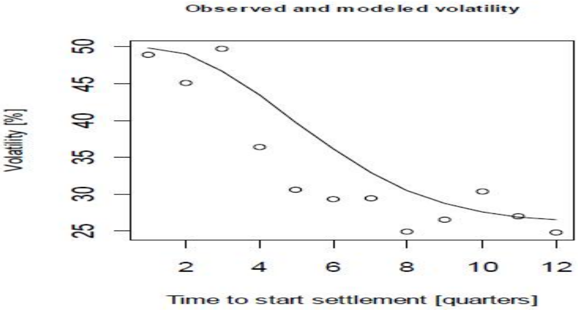
- Spikes lead to highly leptokurtic spot price returns
- Example with UK electricity returns
  - Seasonality removed
  - Daily returns
  - Normal probability plot



- Returns are distinctively heavy tailed
  - Extreme events have much higher probability than the normal distribution can explain
- Small variations have higher probability than normal
- The effect of spikes....
  - ...but maybe also stochastic volatility?

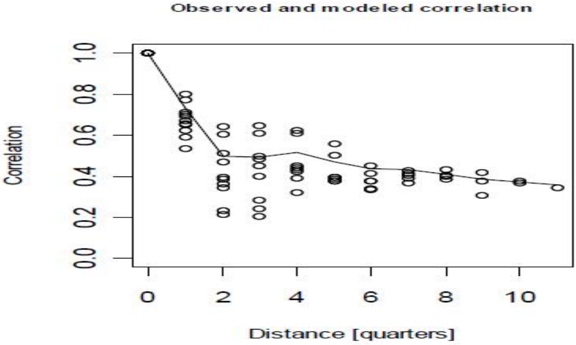
Stylized facts of electricity forwards

- In commodity markets: samuelson effect
  - Volatility of forwards decrease with time to maturity
  - Reflection of the mean-reversion of the forward price
  - The influence becomes insignificant in the long end of the market
- Plot of Nordpool quarterly contracts, empirical volatility

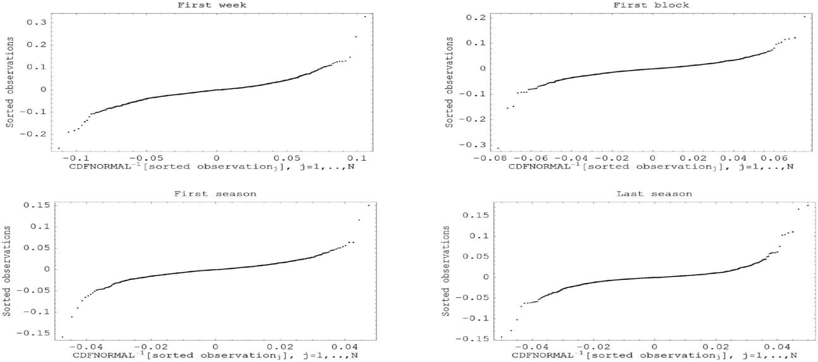


- How are different contracts related statistically?
  - Monthly, quarterly, yearly....
  - Different times to maturity
- Can we explain most of the uncertainty by a few factors?
  - Recall fixed-income markets: PCA indicate $\sim$ 3 factors for explaining about 95-99% of the uncertainty
  - Electricity different!
- Koekebakker and Ollmar 2005: 10 factors not enough to capture 95% of the uncertainty in the forward market
  - A lot of idiosyncratic risk
  - Points towards random field models

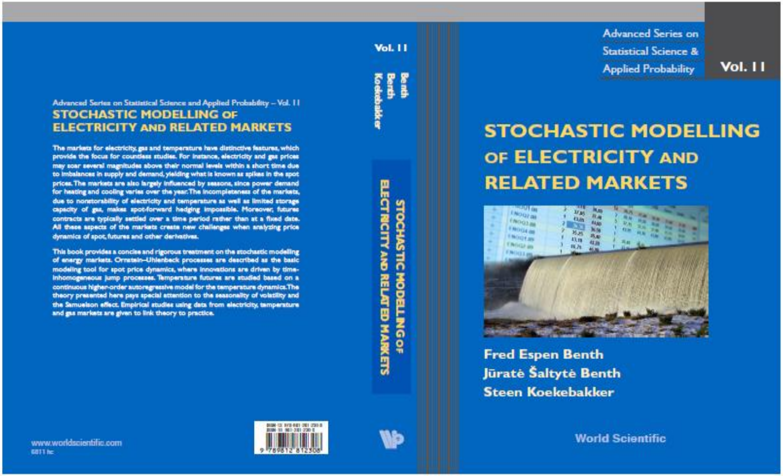
- Study of the correlation structure of quarterly contracts in NordPool (Andresen, Westgaard and Koekebakker 2009)



- Logreturns of forward prices are not normally distributed
  - Same shape as for stock price returns
  - heavy tailed



Commercial break.......



Stochastic modelling of electricity prices

- Situation similar to that of fixed-income markets
- Spot price $\leftrightarrow$ short rate of interest
    - Cannot create portfolios in the spot
- Forward contracts $\leftrightarrow$ forward rates
    - ... or at least zero-coupon bonds
- Modelling problem:
    - Spot modeling, to price forwards
    - What is the link between spot and forwards?
    - HJM-approach, that is, direct modeling of forward prices

Spot price modeling

- Building blocks for electricity spot price models are mean-reversion processes
  - AR(1)-processes in continuous time

$$dX(t) = -\alpha X(t)\,dt + dL(t)$$

- dL is a Lévy/Sato process
  - $dL = \sigma dB$ – a Brownian motion
  - $dL = ZdN$ – a compound Poisson process
  - or a mixture of both, NIG say
- Explicit form

$$X(t) = X(0)\mathrm{e}^{-\alpha t} + \int_0^t \mathrm{e}^{-\alpha(t-s)}\,dL(s)$$

- Levy semimstationary (LSS) process

- Basic example: One-factor Schwartz model with jumps

$$S(t) = \Lambda(t)\mathrm{e}^{X(t)} \quad dX(t) = -\alpha X(t)\,dt + \sigma\,dB(t) + Z\,dN(t)$$

- More relevant: multifactor dynamics, with stochastic volatility (B. and Vos 2009)

$$S(t) = \Lambda(t)\exp\left(\sum_{i=1}^{n} X_i(t)\right)$$

$$dX_i(t) = -\alpha_i X_i(t)\,dt + \sigma_i(t)\,dL_i(t)$$

- σ_i again a LSS process, modulated by subordinators to ensure positivity
    - BNS stochastic volatility model

- Additive model (B., Kallsen and Meyer-Brandis 2007)

$$S(t) = \Lambda(t) \sum_{i=1}^{n} X_i(t)$$

- $X_i(t)$ LSS processes, driven by subordinators
  - spot price positive
- Stable CARMA(2,1) model (Garcia, Klüppelberg and Müller 2009)

$$S(t) = \Lambda(t) + Y(t)$$

where

$$Y(t) = \int_{(-\infty,t]} g(t-s)\, dL(s) \qquad g(u) = \mathbf{b}' \mathrm{e}^{Au} \mathbf{e}_p$$

- L is a stable process

- All models above can be embedded into a general class of LSS processes

$$S(t) = \Lambda(t)\exp(X(t)) \qquad S(t) = \Lambda(t) + X(t)$$

where

$$X(t) = \int_{-\infty}^{t} g(t-s)\sigma(s)\,dL(s)$$

- Volatility σ again an LSS process
- Note: X is in general *not* a semi-martingale
  - ...but that is not a problem, the spot is not tradeable in any case....

Forward pricing in electricity markets based on spot

- Requires a pricing measure, risk neutral probability Q
  - Some equivalent measure Q, no martingale condition required
- Usually, one uses an Esscher transform (or Girsanov for Brownian models)
  - Based on the MGF of L
  - Market price of risk
- Parametric approach, where the market price of risk must be estimated
  - From observed forward prices, say

- Additive model suitable for pricing forwards delivering over a period (like electricity contracts)
- Suppose for technical simplicity $\Lambda(t) = 1$
- Consider forward contract delivering the spot $S(t)$ over the time interval $[T_1, T_2]$
- Forward price at time $t \leq T_1$

$$F(t, T_1, T_2) = \mathbb{E}_Q \left[\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} S(u)\, du \middle| \mathcal{F}_t \right]$$

- Note the averaging: due to the denomination of forward prices in MWh, and not MW!

- The forward price is explicitly computable

$$F(t,T_1,T_2)=\sum_{i=1}^{n}\overline{\alpha}_i(t,T_1,T_2)X_i(t)+\frac{\mathbb{E}_Q[L_i(1)]}{\alpha_i}\left(1-\overline{\alpha}_i(t,T_1,T_2)\right)$$

- $\overline{\alpha}_i$ is the average of $\exp(-\alpha_i(u-t))$ for $u\in[T_1,T_2)$

$$\overline{\alpha}_i(t,T_1,T_2)=\frac{e^{-\alpha_i(T_1-t)}-e^{-\alpha_i(T_2-t)}}{\alpha_i(T_2-T_1)}$$

- Average Samuelson effect
    - $\exp(-\alpha_i(u-t))$ increasing when time to maturity $u-t$ goes to zero
    - "Volatility" goes up as we approaches delivery at time u

- The dynamics of the forward can be computed (using Esscher transform pricing measure)

$$F(t, T_1, T_2) = F(0, T_1, T_2) + \sum_{i=1}^{n} \int_0^t \overline{\alpha}_i(s, T_1, T_2)\, dL_i^{\theta}(s)$$

- L_i^{θ} is L_i under the Esscher transform using parameter θ
  - L_i being the subordinator driving X_i
- Note the integral form of the forward price
  - Stochastic integral of a function depending on time and the delivery period $[T_1, T_2]$
- Exponential models cannot be computed in general for contracts with delivery period

- Additive spot gives a forward price with finite factors
  - Not able to model idiosyncratic risk for each contract
- Reasonable generalization by ambit fields
- Simplified setting, using fixed-delilvery contracts $F(t, T)$
  - Musiela parametrization: $f(t,x) \triangleq F(t,t+x)$, x being *time-to-maturity*

$$f(t,x) = \int_{\mathcal{A}_t(x)} q(t,x;s,y)\sigma(s,y)\, L(dsdy)$$

- L is a Lévy basis, $\mathcal{A}_t(x)$ is an ambit set, q a deterministic function, σ a volatility field (again given as an ambit field)

- Ambit field model provides flexibility in
  - modeling the Samuelson effect through q
  - Including heavy-tailed return distributions using L
  - Complex dependency structures among contracts using $\mathcal{A}_t(x)$ and L
    - and second-order dependencies through σ
- Problem: not in general semi-martingales
  - Conditions required to avoid arbitrage dynamics

Conclusions

- Presented how electricity markets function
- Discussed some of the stylized facts of prices in these markets
- Motivated the use of ambit processes for electricity price modeling
  - Both spot and forwards
- Next on the agenda:
  - Properties of LSS spot models
  - Fitting of such to data
  - Applications to forward pricing
  - HJM modeling using ambit fields
  - Properties of such
  - Calibration to data
  - ....

Almut will tell us all about this in the next talk!

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