Modelling electricity spot and forward prices by ambit fields

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Joint work with

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- Aim: Modelling electricity spot and forward prices in a general framework which accounts for the stylised facts.
- Stylised facts: mean-reversion, (non-)stationarity, stochastic volatility, jumps, Samuelson effect.
- Main innovations:
  - Model the spot price directly (in stationarity) — not its dynamics.
  - Use Lévy semistationary processes and ambit fields as building blocks.

We proceed as follows. We

- model electricity spot prices by Lévy semistationary processes,
- model electricity forward prices by ambit fields,
- establish the link between spot and forward prices in our new modelling framework.

We use the class of Lévy semistationary ($\mathcal{LSS}$) processes as a building block for our new model for electricity spot prices.

A $\mathcal{LSS}$ process $Y = \{Y_t\}_{t\in\mathbb{R}}$ is given by

$$Y_t = \mu + \int_{-\infty}^{t} g(t-s)\omega_s \mathrm{d}L_s + \int_{-\infty}^{t} q(t-s)a_s \mathrm{d}s, \qquad (1)$$

where μ is a constant, L is a Lévy process, g and q are nonnegative deterministic functions on $\mathbb{R}$, with $g(t) = q(t) = 0$ for $t \leq 0$, and ω and a are càdlàg processes.

Note:

- The name Lévy semistationary processes has been derived from the fact that the process Y is stationary as soon as ω and a are stationary.
- The integration in (1) is in the Itô sense.

For the electricity spot price model we ignore the ‘drift’, hence

$$Y_t = \int_{-\infty}^{t} g(t-s)\omega_s \mathrm{d}L_s. \tag{2}$$

Furthermore we assume that ω and L are *independent.*

Remark A spot model based on (2) accounts for jumps, stochastic volatility, stationarity and the Samuelson effect in a very general way.

Using Y as a building block, we can now formulate a *geometric* and an *arithmetic* model for the electricity spot price.

Let $\Lambda : \mathbb{R}_+ \to \mathbb{R}_+$ denote a bounded and measurable deterministic seasonal function.

In a geometric set up, we define the spot price $S^g = (S_t^g)_{t\in\mathbb{R}}$ by

$$S_t^g = \Lambda(t)\exp(Y_t), \qquad Y_t = \int_{-\infty}^{t} g(t-s)\omega_s \mathrm{d}L_s. \tag{3}$$

Special cases of such models include the classical Schwartz model and the CARMA–based models.

Alternatively, one can construct a spot price model which is of arithmetic type.

The following condition is sufficient for price positivity.

Assumption (P): Let L be a Lévy subordinator and let the kernel function g in (2) be positive.

If assumption (P) is satisfied, we define the electrictity spot price $S^a = (S_t^a)_{t\geq 0}$ by

$$S_t^a = \Lambda(t) Y_t, \qquad Y_t = \int_{-\infty}^{t} g(t-s)\omega_s \mathrm{d}L_s. \tag{4}$$

Note that a special case of a superposition of such a model is given by the model in Benth, Kallsen, & Meyer–Brandis (2007).

Extensions: Superposition; non–stationary components, leverage effect etc..

The forward price $F_t(T)$ at time t for contracts maturing at time $T \geq t$ is given by

$$F_t(T) = \mathbb{E}_Q\left[S_T^g \,|\, \mathcal{F}_t\right] ,$$

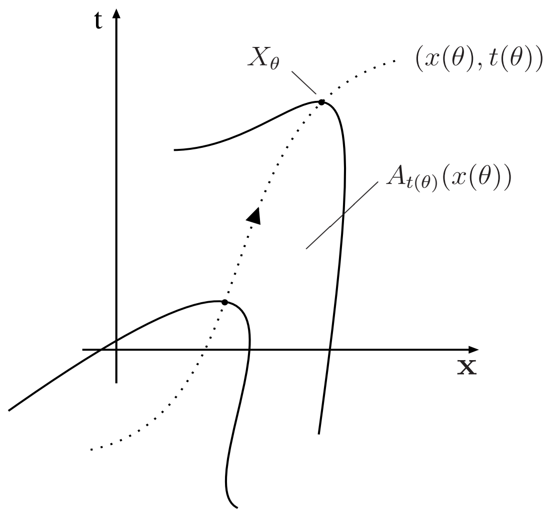
with Q being an equivalent probability to P.

When we compute that, we observe that (under some regularity conditions) $ln(F_t(T))$ is given by a superposition of *ambit fields*.

We think of ambit fields as being of the form

$$Y_t(x) = \mu + \int_{A_t(x)} g(\xi, s; x, t)\, \sigma_s(\xi)\, L(\mathrm{d}\xi, \mathrm{d}s) + \int_{D_t(x)} q(\xi, s; x, t)\, a_s(\xi)\, \mathrm{d}\xi \mathrm{d}s,$$

where $A_t(x)$, and $D_t(x)$ are ambit sets, g and q are deterministic function, $\sigma \geq 0$ is a stochastic field referred to as *volatility*, and L is a *Lévy basis*. (Integration in the sense of Rajput & Rosinski (1989).)



Note that a $\mathcal{LSS}$ process is the null–spatial case of an ambit field.

- We assume that $\sigma \perp\!\!\!\perp L$.
- Let $t \geq 0$ denote the current time, $T > 0$ the time of maturity of the forward contract and $x = T - t$ the corresponding time to maturity.

We suggest to model the forward price as a process, which is stationary in time t. In order to ensure that, we make the following structural assumptions:

- The damping function satisfies $h(\xi, s, x, t) = k(\xi, t - s, x)$, for a function k,
- $\sigma_s(\xi)$ is stationary in s,
- the ambit set is of the form $A_t(x) = A_0(x) + (0, t)$.

We propose to model a forward contract, denoted by $f_t(x)$ by

$$f_t(x) = \int_{A_t(x)} k(\xi, t - s; x)\sigma_s(\xi)L(d\xi, ds),$$

where all quantities are specified as before.

Remark

- We use a tempo–spatial model to model the forward which depends on the *current time* t and the *time to maturity* $x = T - t$ (this is the spatial component).
- Such a model allows for stationarity in time, stochastic volatility, Samuelson effect etc.

How shall we choose σ, $A_t(x)$, k etc.?

A very general specification would be the following one. Let

$$\sigma_t^2(x) = \int_{C_t(x)} j(\xi, s; x, t) L(d\xi, ds),$$

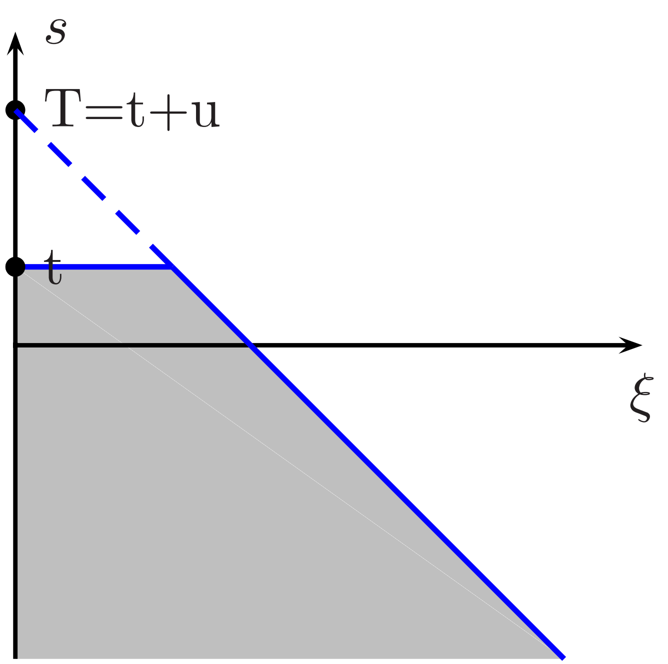
for a Lévy basis L, a deterministic kernel function j and an ambit set $C_t(x)$.

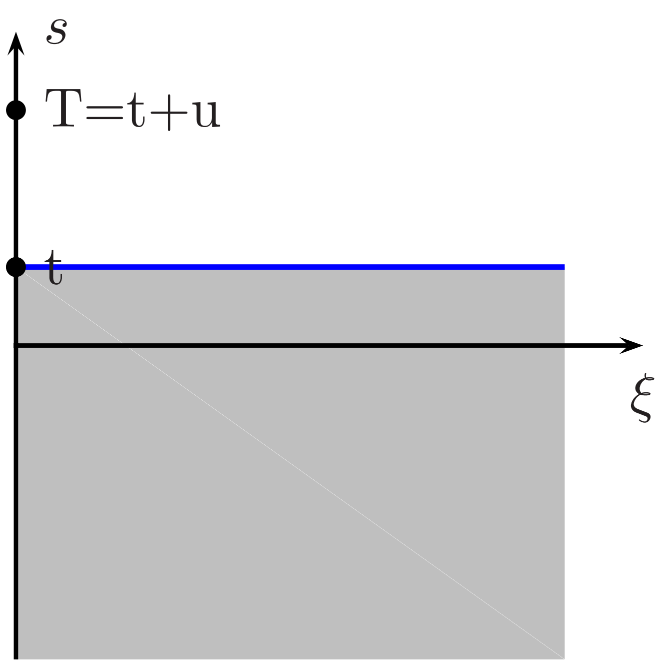
In order to ensure that forward contracts close in maturity dates are strongly correlated with each other, we could choose the Lévy kernel j such that

$$Cor(\sigma_t^2(x), \sigma_t^2(\bar{x}))$$

is high for values of x and $\bar{x}$ which are close to 0 (i.e. closeness to maturity).

Let $u = T - t$ and $A_t^{(c)}(u) = \{(\xi, s) : s \leq t, 0 \leq \xi \leq c(t + u - s)\}$.





Aim: We wish to have (at least in law)

$$\lim_{x \downarrow 0} f_t(x) \to f_T(0) = Y_T, \qquad \text{where} \quad Y_t = \int_{-\infty}^{t} g(t-s)\omega_s dZ_s.$$

Now we consider the special case of a standard normal Lévy base L and a standard Brownian motion Z.

Then, f is mixed normal, in particular

$$f_t(x)|\sigma \sim N\left(0, \int_{A_t(x)} k(\xi, t-s, x)^2 \sigma_s^2(\xi)\, d\xi ds\right).$$

So, for an ambit set given by
$A_t(x) = \{(\xi, s) : s \leq t, 0 \leq \xi \leq c(t + x - s)\}$, we get for the conditional variance of f given σ that

$$\int_{-\infty}^{t} \int_{0}^{c(t+x-s)} k(\xi, t-s, x)^2 \sigma_s^2(\xi)\, d\xi ds = \int_{0}^{\infty} \int_{0}^{c(v+x)} k(\xi, v, u)^2 \sigma_{t-v}^2(\xi) d\xi dv$$

$$\rightarrow \int_{0}^{\infty} \int_{0}^{cv} k(\xi, v, 0)^2 \sigma_{T-v}^2(\xi) d\xi dv, \quad \text{as } x \rightarrow 0.$$

In the specific case when $c = \infty$, k factorises as in
$k(\xi, v, 0) = k_0(v)\, k_1(\xi)$ and $\omega_{v,t-v}^2 \stackrel{L}{=} \int_0^\infty k_1(\xi)^2 \sigma_{t-v}^2(\xi) d\xi$, we get
$g = k_0$ and

$$f_t(x) \rightarrow Y_T, \quad \text{as } x \rightarrow 0, t \rightarrow T.$$

- We propose to model
  - electricity spot prices by Lévy semistationary processes and
  - electricity forward prices by ambit fields.
- We establish a link between the spot and the forward model.

The next steps:

- Empirical work: How can we estimate/simulate from the new models? Which choice of the kernel functions k, g mimics the Samuelson effect in a realistic way? What are good choices for the stochastic volatility field? etc.
- Theoretical work: Can we establish a stochastic calculus for ambit processes? etc.